1 Derive continuity equation for regolith thickness

$$H(x, y, t) = H_0(x, y) + U(x, y, t) + V(x, y, t, H) \tag{1}$$

Where H_0 is the initial topography,

U is the vertical component of tectonic displacement,

and V is the geomorphic displacement.

Because U is the same for H and B at a given point (x, y), it does not change the regolith thickness:

$$h(x, y) = (H(x, y) - U(x, y)) - (B(x, y) - U(x, y))$$

or

$$h(x, y) = H(x, y) - B(x, y)$$

Therefore, it is convenient to consider soil continuity in terms of soil thickness.

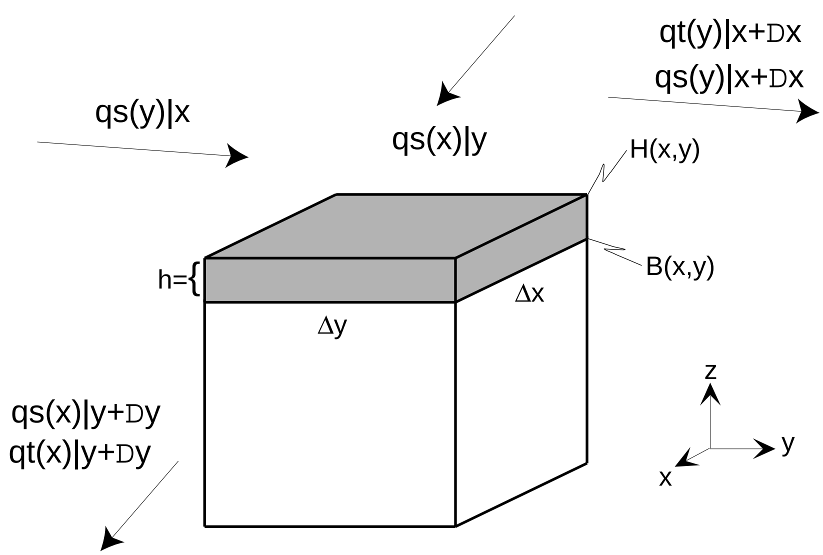

Figure 1: regolith continuity for a hillslope element

We use the derivation of conservation of transportable (regolith) for our control area (note that is a 2 dimensional problem):

$$\begin{aligned}&\Big\{\text{net rate of mass efflux from control area}\Big\} + \\ &\Big\{\text{net rate of mass accumulation in control area}\Big\} - \\ &\Big\{\text{rate of production of mass within control area}\Big\} \\ &= 0\end{aligned}$$

So let's take each term and evaluate it:

1. Net rate of mass efflux from control area. Consider the mass flux vector

$$\vec{n} = \rho\vec{v} = \rho_s\vec{q}_s \qquad [ML^{-3} \cdot L^3 \cdot T^{-1}] \tag{2}$$

$$\begin{aligned}\text{where} \rho_s &\text{is sediment density} [ML^{-3}] \\ \vec{q}_s &\text{is sediment volume flux} [L^3T^{-1}]\end{aligned}$$

If we resolve the mass vlux vector into its cartesian components, we have:

in the x direction: $n_x\Delta y|_{x+\Delta x} - n_x\Delta y|_x$

in the y direction: $n_y\Delta x|_{y+\Delta y} - n_y\Delta x|_y$

in the z direction: $n_z = 0$

2. Net rate of accumulation of mass in the control area:

$$\rho_s\frac{\partial h}{\partial t}\Delta x\Delta y \tag{3}$$

3. Net rate of production of mass in the control area:

$$-\rho_r\frac{\partial B}{\partial t}\Delta x\Delta y \qquad \rho_r \text{is the rock density} \tag{4}$$

Put it all together:

$$\begin{aligned}\Big\{(n_x\Delta y|_{x+\Delta x} - n_x\Delta y|_x) + (n_y\Delta x|_{y+\Delta y} - n_y\Delta x|_y)\Big\} &+ \\ \rho_s\frac{\partial h}{\partial t}\Delta x\Delta y &- \\ -\rho_r\frac{\partial B}{\partial t}\Delta x\Delta y &= 0\end{aligned}$$

Divide through by $\Delta x \Delta y$ and simplify

$$\begin{aligned}
(\frac{\Delta n_x}{\Delta x} + \frac{\Delta n_y}{\Delta y}) + \rho_s \frac{\partial h}{\partial t} + \rho_r \frac{\partial B}{\partial t} &= 0 \\
(\frac{\partial}{\partial x} n_x + \frac{\partial}{\partial y} n_y) + \rho_s \frac{\partial h}{\partial t} + \rho_r \frac{\partial B}{\partial t} &= 0 \\
\nabla \vec{n} + \rho_s \frac{\partial h}{\partial t} + \rho_r \frac{\partial B}{\partial t} &= 0 \\
\rho_s \nabla \vec{q}_s + \rho_s \frac{\partial h}{\partial t} + \rho_r \frac{\partial B}{\partial t} &= 0 \qquad (5)
\end{aligned}$$

Thus (5) is the continuity equation for regolith thickness.

2 Evaluation of dimensions

If we look again at the first portion of (5),

$$\begin{aligned}
(\frac{\Delta n_x}{\Delta x} + \frac{\Delta n_y}{\Delta y}) + \rho_s \frac{\partial h}{\partial t} + \rho_r \frac{\partial B}{\partial t} &= 0 \\
\rho_s(\frac{\Delta q_{sx}}{\Delta x} + \frac{\Delta q_{sy}}{\Delta y}) + \rho_s \frac{\partial h}{\partial t} + \rho_r \frac{\partial B}{\partial t} &= 0 \\
\text{Focusing on the first term:} & \\
\rho_s(\frac{\Delta q_{sx}}{\Delta x} + \frac{\Delta q_{sy}}{\Delta y}) &= \rho_s \nabla \vec{q}_s \\
\text{Dimensionally:} & \\
[ML^{-3}](\frac{[L^3T^{-1}]}{[L]} + \frac{[L^3T^{-1}]}{[L]}) & \\
[ML^{-3}]([L^2T^{-1}] + [L^2T^{-1}]) & \\
[ML^{-3}][L^2T^{-1}] &= \rho_s \nabla \vec{q}_s \\
\text{So, indeed} & \\
[L^2T^{-1}] &= \nabla \vec{q}_s
\end{aligned}$$